3 (Sem–2/CBCS) CSC HC 2

2023

COMPUTER SCIENCE

(Honours Core)

(Discrete Structure)

Paper : CSC–HC–2026

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 10 = 10$

   (a) Isolated vertices are vertices with _____ degree. *(Fill in the blank)*

   (b) If the cardinalities of two sets are same, they are called _____ sets. *(Fill in the blank)*

   (c) A relation R on set A is called _____ relation if xRy implies yRx, $\forall x \in A$ and $\forall y \in A$. *(Fill in the blank)*

(d) In any tree (with two or more vertices), there are at least _____ pendant vertices. *(Fill in the blank)*

(e) What is the correct translation of the following statement into mathematical logic ?

"Some real numbers are rational."

(i) $\exists x$ (real $(x) \vee$ rational (x))

(ii) $\forall x$ (real $(x) \rightarrow$ rational (x))

(iii) $\exists x$ (real $(x) \wedge$ rational (x))

(iv) $\forall x$ (rational $(x) \rightarrow$ real (x))

(Choose the correct option)

(f) The correct recursive definition for sequence $1, 5, 5^2, 5^3, \cdots$ is

(i) $a_1 = 1 ; a_k = 5a_k$, for $k \geq 1$

(ii) $a_1 = 1 ; a_{k+1} = 5a_k$, for $k \geq 1$

(iii) $a_1 = 1 ; a_k = 5a_k^2$, for $k \geq 1$

(iv) $a_1 = 1 ; a_{k+1} = 5a_{k+1}$, for $k \geq 1$

(Choose the correct option)

(g) What is 'contradiction' in logic ?

(h) Define the Big omega (Ω) notation.

(i) What is 'tautology' ?

(j) What is 'simple graph' ?

2. Define the following terms : 2×5=10

(i) Disjoint set

(ii) Recurrence tree

(iii) Asymptotic notation

(iv) Planar graph

(v) Spanning tree

3. Answer ***any four*** of the following : 5×4=20

(a) What is pigeonhole principle ? If 7 colours are used to point 50 bicycles then *at least* how many bicycles will be of same colour.

(b) Using the principle of mathematical induction prove that for any positive integer n, $6^n - 1$ is divisible by 5.

(c) What do you mean by normal form in logic ? Obtain the principal disjunction normal form (PDNF) for the formula

$$(\sim p \rightarrow \gamma) \wedge (q \leftrightarrow p)$$

(d) Define 'rank' and 'nullity' of a graph. Calculate the rank and nullity for the graph in *Fig.* 1 :

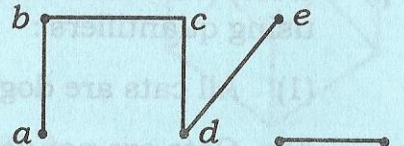


Fig. 1

(e) Solve the recurrence relation

$a_n = 3a_n - 1,\ n \geq 1,$ given $a_0 = 1$

(f) Define the principle of inclusion and exclusion for two sets A and B.

If $|A \cup B| = 12$, $A \subseteq B$ and $|A| = 3$

then calculate the value of $|B|$.

4. Answer ***any four*** of the following : 10×4=40

(a) (i) Define a partial order relation.

If a relation

$R = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 3)\}$

on set $A = \{1, 2, 3\}$, determine whether R is partial order relation or not ? 5

(ii) A team of four players has to be selected from six boys and four girls. How many different ways a team can be selected, if *at least one* boy must be there in the team ?

5

(b) (i) Write the following statements using quantifiers : 5

(1) All cats are dogs.

(2) Cats are not dogs.

(3) Not all cats are dogs.

(4) There are cats that are not dogs.

(5) Something is either a cat or a dog.

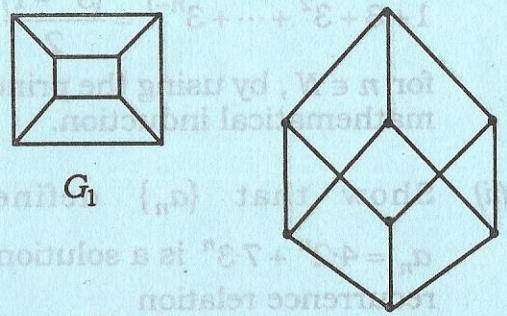
(ii) Define the logical equivalent. Show that 5

$$(P \to Q) \wedge \left(P \to \tilde{Q}\right) \equiv \tilde{P}$$

(c) (i) What is Hamiltonian graph ? Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit. 5

(ii) What is isomorphism of two graphs ? Are the two graphs G_1 and G_2 in *Fig.* 2 isomorphic ? Why ?

5



G_1 G_2

Fig. 2

(d) (i) What is a valid argument ? Use *any* method to show the following argument is valid : 5

$$\frac{p \quad \sim q \leftrightarrow \sim p}{\therefore q}$$

(ii) What is spanning tree ? Prove that every corrected graph has *at least one* spanning tree. 5

(e) (i) Define the asymptotic notation in detail. Calculate the Big (O), Big (Ω) and Big (Θ) for

$f(n) = 5n^2 + n$. 5

(ii) Define the countable infinite set and uncountable infinite set with proper example. 5

(f) (i) Show that

$$1 + 3 + 3^2 + \cdots + 3^{n-1} = \frac{(3^n - 1)}{2},$$

for $n \in N$, by using the principle of mathematical induction. 5

(ii) Show that $\{a_n\}$ defined by $a_n = 4 \cdot 2^n + 7 \cdot 3^n$ is a solution of the recurrence relation

$a_n - 5a_{n-1} + 6a_{n-2} = 0$ 5

(g) (i) Define the Kuratowski's two graphs k_5 and $k_{3,3}$ with proper diagram. 5

(ii) Prove that, 'the complete graph of five vertices is nonplanary'. 5

(h) (i) Define the 'symmetric difference' and 'set difference' of two sets (by using Venn diagram representation). 4

(ii) If $A = \{1, 2, 3\}$, $B = \{5, 6\}$, $C = (2, 3)$, then find—

(a) $(C \times B) - (A \times B)$

(b) $A \oplus B \oplus C$ 6
