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**3 (Sem-3/CBCS) PHY HC 1**

**2023**

**PHYSICS**

(Honours Core)

Paper : PHY-HC-3016

**(Mathematical Physics-II)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions:  $1 \times 7 = 7$

(a) Define the ordinary point of a second order differential equation.

(b) Show that  $P_0(x) = 1$ .

(c) Write the Laplace equation in spherical polar co-ordinate system.

(d) A partial differential equation has

(i) one independent variable

Contd.

- (ii) more than one dependent variable
- (iii) two or more independent variables
- (iv) no independent variable.

(Choose the correct option)

- (e) If  $A$  and  $B$  are two square matrices of order  $n$ , show that

$$\text{Trace}(A + B) = \text{Trace } A + \text{Trace } B$$

- (f) Which one of the following is the value of  $\Gamma\left(\frac{1}{2}\right)$  ?

(i)  $\sqrt{\pi/2}$

(ii)  $\sqrt{\pi}$

(iii)  $\pi$

(iv)  $\sqrt{\pi}/2$

- (g) Define self adjoint of a matrix.

2. Answer the following questions:  $2 \times 4 = 8$

- (a) Check the behaviour of point  $x = 0$  for the differential equation

$$\frac{d^2 y}{dx^2} - \frac{6}{x} y = 0$$

- (b) If  $\int_{-1}^{+1} P_n(x) dx = 2$ , find the value of  $n$ .

- (c) Express the Fourier series in complex form.

- (d) Verify the matrix

$$A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix} \text{ is a unitary}$$

matrix.

3. Answer **any three** of the following questions:  $5 \times 3 = 15$

- (a) Find the power series solution of the following differential equation :

$$(1 - x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

about  $x = 0$ .

- (b) Define Gamma function. Show that

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \quad 1 + 4 = 5$$

(c) Establish the following recurrence formula for Legendre polynomial  $P_n(x)$

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

(d) Show that the Fourier expansion of the function  $f(x) = x$ ,  $0 < x < 2\pi$  is

$$x = \pi - 2 \left[ \sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x + \dots \right]$$

(e) What is eigenvalue of a matrix? Show that the eigenvalues of Hermitian matrix are real. 1+4=5

4. Answer **any three** of the following questions : 10×3=30

(a) (i) If  $P_n(x)$  be the polynomial of Legendre's differential equation, show that

$$P_n(x) = \frac{1}{2^n \cdot n!} \cdot \frac{d^n}{dx^n} (x^2 - 1)^n \quad 6$$

(ii) Prove that 4

$$\int_{-1}^{+1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$$

(b) (i) What is Beta function? Show that

$$(a) \beta(1, 1) = 1$$

$$(b) \beta(m, n) = \beta(n, m) \quad 1+1+3=5$$

(ii) If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , show that

$A^2 - 4A - 5I = 0$  where  $I$  and  $O$  are the unit matrix and the null matrix of order 3 respectively. Also use this result to find  $A^{-1}$ . 3+2=5

(c) Find the Fourier series expansion of

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Also show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad 6+4=10$$

(d) (i) Diagonalize the following matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad 5$$

- (ii) Show that the generating function for Hermite polynomial  $H_n(x)$ , for integral  $n$  and real value of  $n$  is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x) \quad 5$$

- (e) (i) Write the one dimensional diffusion equation (heat flow equation) and find the general solution of the same by the method of separation of variable.

1+7=8

- (ii) For the Pauli spin matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \text{ and}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ show that}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3 \quad 2$$

- (f) (i) Write the Orthogonality conditions of *sine* and *cosine* functions.

1½+1½=3

- (ii) A square wave function is represented as

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x < 0 \\ h, & \text{for } 0 \leq x < \pi \end{cases}$$

Draw the graphical representation of the wave function and expand the same in Fourier series.

1+6=7