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3 (Sem–5/CBCS) PHY HE 3

2023

PHYSICS

(Honours Elective)

Paper : PHY–HE–5036

(Advanced Mathematical Physics – I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) What is isomorphism in case of a vector space ?

   (b) Define associated tensor.

   (c) What is field ? Give *two* examples.

   (d) State quotient law of tensors.

   (e) Write the scalar triple product $\vec{A}.\left(\vec{B}\times\vec{C}\right)$ using tensor notation.

Contd.

(f) What is Moment of Intertia tensor ?

(g) If $A=\begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}$, find 2^A.

2. Answer the following : 2×4=8

(a) Show that diagonalizing matrix of a symmetric matrix is orthogonal.

(b) Show that the vectors $W_1=[2,\ 1,\ 1]$, $W_2=[-2,\ 1,\ 2]$ and $W_3=[0,\ 0,\ 1]$ are linearly independent.

(c) What is Minkowski space ? Define a four vector in this space.

(d) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

3. Answer ***any three*** of the following question : 5×3=15

(a) What is binary operation ? Determine the identity element and inverse for the following binary operation :

$(a,b)*(c,d)=(ac,\ bc+d)$. 1+4=5

(b) Diagonalize the matrix

$$A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}.$$

(c) (i) If a contravariant tensor of rank two is symmetric in one co-ordinate system, show that it is symmetric in any co-ordinate system. 3

(ii) If A_λ is a covariant tensor of rank one, verify whether $\dfrac{\partial A_\lambda}{\partial x^\mu}$ is a tensor or not. 2

(d) (i) Find the number of independent components of a second rank symmetric tensor in n-dimensional space. 2

(ii) Using the relation $ds^2=g_{ij}\,dx^i\,dx^j$, prove that g_{ij} is a symmetric tensor. 3

(e) Using tensor-analysis, show that : 2+3=5

(i) $\varepsilon_{ils}\,\varepsilon_{mls} = 2\delta_{im}$

(ii) $\vec{\nabla}.\vec{A}$ is an invariant.

4. Answer **any three** of the following questions : 10×3=30

(a) (i) Define basis and dimension of a linear vector space. If x, y, z are linearly independent vectors, determine whether the vector $x+y,\ y+z$ and $z+x$ are linearly dependent or not. 2+3=5

(ii) Use ε_{ijk} to find the vector associated with the following anti-symmetric tensor of rank two :

$$\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

and to express cross product of vectors $\vec{A}$ and $\vec{B}$. 3+2=5

(b) (i) What is Group ? Check whether the set I of all integers with the binary operation $*$ defined by

$a * b = a + b + 1$ forms a Group.

1+4=5

(ii) Show that every linearly independent vector belonging to a vector space has a unique representation as a linear combination of its bases vector. 5

(c) (i) Using tensor analysis prove the following vector identities : 2+2+3=7

(a) $\vec{\nabla}.\left(\vec{\nabla}\times\vec{A}\right)=0$

(b) $\vec{\nabla}\times\left(\phi\vec{A}\right)=\phi\left(\vec{\nabla}\times\vec{A}\right)+\vec{\nabla}\phi\times\vec{A}$

(c) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}$

(ii) Find the second order antisymmetric tensor associated with the vector $2\hat{i}-3\hat{j}+\hat{k}$. 3

(d) (i) Solve the coupled linear differential equations using matrix method :

$$y_1' = 2y_1 + 3y_2$$
$$y_2' = 4y_1 + y_2$$

where $y_1(0) = 2,\ y_2(0) = 1$. 5

(ii) Show that in Cartesian co-ordinate system, the contravariant and covariant components of a vector are identical. 5

(e) What is matric tensor g_{qr} ? Calculate the co-efficients of matric tensor in spherical polar co-ordinate and then write the matric tensor. Prove that the matric tensor g_{qr} is a symmetric co-varient tensor of order 2. 2+2+6=10

(f) (i) State Hooke's law in elasticity using tensor notation. If ε_{ij}'s denote fractional deformation, establish the relation,

$\delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$, where δ is the change in volume associated with the deformation. 2+5=7

(ii) Prove that eignevalues of a hermitian matrix are real. 3

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