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3 (Sem–5/CBCS) MAT HE 4/5/6

2023

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

Full Marks : 80

Time : Three hours

OPTION–B

Paper : MAT–HE–5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer: $1 \times 10 = 10$

(i) The general LPP is in standard form if

(a) the constraints are inequalities of $\leq$ type

(b) the constraints are inequalities of $\geq$ type

(c) the constraints are strict equalities

(d) the decision variables are unrestricted in sign

(ii) If a given LPP has two feasible solutions, then

(a) it cannot have infinite number of feasible solutions

(b) it has infinite number of feasible solutions

(c) it has no basic feasible solution

(d) the LPP must have an unbounded solution

(iii) The LPP

$$\text{Maximize } x_1 + x_2$$

$$\text{subject to } \begin{aligned} x_1 - x_2 &\geq 1 \\ -x_1 + x_2 &\geq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(a) has no feasible solution

(b) has infinitely many optimal solutions

(c) has unbounded solution

(d) has unique optimal solution

(iv) Choose the correct statement :

(a) The maximum number of basic solutions of a system $AX = b$ of m equations in n unknowns $(n > m)$ is $m + n - 1$

(b) For the solution of any LPP by simplex method, the existence of an initial basic feasible solution is always assumed

(c) When the constraints are of $\geq$ type, artificial variables are introduced to convert them into equalities

(d) In phase I of the two-phase method, the sum of the artificial variables is maximized subject to the given constraints to obtain a basic feasible solution to the original LPP *(v)* If the primal problem has a finite optimal solution, then the dual problem

(a) also has a finite optimal solution

(b) has an unbounded solution

(c) has no feasible solution

(d) has no basic feasible solution

(vi) For optimal feasible solutions of the primal and dual systems, whenever the i^{th} variable is strictly positive in either system,

(a) the i^{th} variable of its dual is unrestricted in sign

(b) the i^{th} variable of its dual vanishes

(c) the i^{th} relation of its dual is a strict inequality

(d) the i^{th} relation of its dual is an equality

(vii) The total transportation cost to the non-degenerate basic feasible solution to the following transportation problem

	D_1	D_2	D_3	
O_1	14	11	7	5
O_2	13	15	7	15
O_3	10	16	7	9
	15	6	8	

obtained by using North-West corner rule is

(a) 249

(b) 294

(c) 318

(d) 347

(viii) In an assignment problem, if a constant is added to or subtracted from every element of a row of the cost matrix $[c_{ij}]$, then

(a) the optimal solution to the assignment problem can never be attained

(b) an assignment which optimizes the total cost for one matrix, also optimizes the total cost for the other matrix

(c) an assignment which optimizes the total cost for the matrix $[c_{ij}]$ does not optimize the total cost for the modified matrix

(d) None of the above

(ix) In a two person zero-sum game, the game is said to be fair if

(a) both the players have equal number of strategies

(b) gain in one player does not match the loss to the other

(c) the value of the game is zero

(d) the value of the game is non-zero

(x) The saddle point of the pay-off matrix

$$\begin{array}{cc} & B \\ A & \begin{array}{|ccc|} \hline 2 & 4 & 5 \\ 10 & 7 & 8 \\ 4 & 5 & 6 \\ \hline \end{array} \end{array}$$

is at

(a) (1, 1)

(b) (2, 2)

(c) (1, 3)

(d) (2, 1)

2. Answer the following questions : 2×5=10

(a) Define hyperplane. Show that a hyperplane is a convex set.

(b) Find a basic feasible solution to the following LPP :

$$\text{Maximize } x_1 + 2x_2 + 4x_3$$
$$\text{subject to } 2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$
$$x_1, x_2, x_3 \geq 0$$

(c) Write the dual of the following LPP :

$$\text{Minimize } 4x_1 + 6x_2 + 18x_3$$

$$\begin{aligned} \text{subject to } & x_1 + 3x_2 \geq 3 \\ & x_2 + 2x_3 \geq 5 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(d) Construct an initial basic feasible solution to the following transportation problem by least cost method :

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	

(e) Give the mathematical formulation of an assignment problem.

3. Answer **any four** of the following : 5×4=20

(a) Examine the convexity of the set

$$S = \left\{ (x_1, x_2) : 3x_1^2 + 2x_2^2 \leq 6 \right\}$$

(b) Use simplex method to show that the LPP

$$\text{Maximize } 2x_1 + x_2$$

$$\begin{aligned} \text{subject to } & x_1 - x_2 \leq 10 \\ & 2x_1 - x_2 \leq 40 \\ & x_1, x_2 \geq 0 \end{aligned}$$

has an unbounded solution.

(c) Show that the dual of the dual is the primal.

(d) Use Vogel's approximation method to obtain an initial basic feasible solution to the transportation problem :

	D_1	D_2	D_3	D_4	
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
	200	225	275	250	

(e) Find the optimal assignment to the assignment problem having the following cost matrix :

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

(f) Define the terms "strategy" and "optimal strategy" with reference to Game theory.

4. Solve the following LPP graphically : 10

Maximize $40x_1 + 35x_2$

subject to $2x_1 + 3x_2 \le 60$

$4x_1 + 3x_2 \le 96$

$8x_1 + 7x_2 \le 210$

$x_1, x_2 \ge 0$

Or

Show that every basic feasible solution of a LPP is an extreme point of the convex set of all feasible solutions of the LPP.

5. Solve the following LPP by simplex method :

Minimize $4x_1 + 8x_2 + 3x_3$

subject to $x_1 + x_2 \ge 2$

$2x_1 + x_3 \ge 5$

$x_1, x_2, x_3 \ge 0$ 10

Or

Use Big-M method to solve the LPP

Maximize $6x_1 + 4x_2$

subject to $2x_1 + 3x_2 \le 30$

$3x_1 + 2x_2 \le 24$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

Is the solution unique ?

6. Solve the dual of the following LPP and write its solution : 10

Maximize $3x_1 - 2x_2$

subject to $x_1 \le 4$

$x_2 \le 6$

$x_1 + x_2 \le 5$

$x_2 \ge 1$

$x_1, x_2 \ge 0$

Or

Solve the following transportation problem :

	D_1	D_2	D_3	D_4	
O_1	3	6	8	5	20
O_2	6	1	2	5	28
O_3	7	8	3	9	17
	15	19	13	18	

7. Solve the following assignment problem : 10

	I	II	III	IV
A	14	42	56	0
B	64	82	91	55
C	44	66	77	33
D	74	90	98	66

Or

For the game with the following pay-off matrix :

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

determine the optimum strategies and the value of the game.

OPTION-B

Paper : MAT-HE-5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (i) How many great circles can be drawn through two given points, when the points are the extremities of a diameter ?

   (ii) Define primary circle.

   (iii) Define polar triangle and its primitive triangle.

   (iv) Define Zenith.

   (v) Explain what is meant by rising and setting of stars.

   (vi) What is the point on the celestial sphere whose latitude, longitude, right ascension and declination, all are zero ?

(vii) Define synodic period of a planet.

(viii) Mention *one* property of pole of a great circle.

(ix) Just mention how a spherical triangle is formed.

(x) What is the declination of the pole of the ecliptic ?

2. Answer the following questions : 2×5=10

(a) In any equilateral triangle *ABC*, show that $2\cos\frac{a}{2}\sin\frac{A}{2}=1$.

(b) Prove that the section of the surface of a sphere made by any plane is a circle.

(c) Discuss the effect of refraction on sunrise.

(d) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(e) Show that right ascension α and declination δ of the sun is always connected by the equation $\tan\delta=\tan\varepsilon\sin\alpha$, ε being obliquity of the ecliptic.

3. Answer ***any four*** questions of the following : 5×4=20

(a) In a spherical triangle *ABC*, prove that

$$\tan\frac{C}{2}=\sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s\sin(s-c)}}.$$

(b) What do you mean by 'rising' and 'setting' of stars ? Derive the relation $\cos H=-\tan\phi\tan\delta$, where the symbols have their usual meanings.

(c) Show that the velocity of a planet in its elliptic orbit is $v^2=\mu\left(\frac{2}{r}-\frac{1}{a}\right)$ where $\mu=G(M+m)$ and a is the semi-major axis of the orbit.

(d) If z_1 and z_2 are the zenith distances of a star on the meridian and the prime vertical respectively, prove that

$\cot\delta=\operatorname{cosec}z_1\sec z_2-\cos z_1$

where δ is the star's declination.

(e) If H be the hour angle of a star of declination δ when its azimuth is A and H' when the azimuth is $(180^\circ + A)$, show that

$$tan\,\phi = \frac{cos\frac{1}{2}(H' + H)}{cos\frac{1}{2}(H' - H)}$$

(f) At a place of latitude ϕ, the declination and hour angle of a heavenly body are δ and H respectively. Calculate its zenith distance z and azimuth A.

4. Answer ***any four*** questions of the following : 10×4=40

(a) In any spherical triangle ABC, prove that $\frac{sin\,A}{sin\,a} = \frac{sin\,B}{sin\,b} = \frac{sin\,C}{sin\,c}$. Also prove that $\frac{sin(A+B)}{sin\,C} = \frac{cos\,a + cos\,b}{1 + cos\,c}$

(b) State Keplar's laws of planetary motion and deduce the differential equation of the path of a planet around the Sun.

(c) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k\,tan\,\xi$, ξ being the apparent zenith distance of a heavenly body. Mention *one* limitation of this formula.

(d) On account of refraction, the circular disc of the sun appears to be an ellipse. Prove it.

(e) Derive the Kepler's equation in the form $M = E - e\,sin\,E$, where M and E are respectively mean anomaly and eccentric anomaly.

(f) Show that the velocity of a planet moving in an ellipse about the sun in the focus is compounded of two constant velocities $\frac{\mu}{h}$ perpendicular to radius vector and $\frac{e\mu}{h}$ perpendicular to major axis.

(g) If the colatitude is C, prove that

$$C = x + \cos^{-1}(\cos x \sec y)$$

where

$\tan x = \cot\delta \cos H, \sin y = \cos\delta \sin H$,

H being the hour angle.

(h) Derive the expressions to show the effect of refraction in right ascension and declination.

OPTION-C

Paper : MAT-HE-5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following : 1×7=7

(a) What are the basic data types associated with C ?

(b) What is the difference between '=' and '==' in C ?

(c) Can a C program be compiled or executed in the absence of a main function ?

(d) Who developed C language ?

(e) What is the output of the program when the value of i is 17 ?

```
#include <stdio.h>

int main ()

{

    int i, k;

    printf ("Enter the value of i:");

    scanf ("%d", &i);

    k = ++i;

    printf ("%d", k);

    return 0;

}
```

(f) 'Intersection' is a reserved word in C. *(True* ***or*** *False)*

(g) What does %5.2 f means in C ?

2. Answer the following : 2×4=8

(a) What is recursion in C ?

(b) What is the difference between the local and global variables in C ?

(c) What are reserved keywords ?

(d) Write the general syntax of scanf () function to read the float variable x.

3. Answer ***any three*** from the following : 5×3=15

(a) Explain with examples the syntax of scanf () and printf () functions.

(b) Draw the flowchart and then write a C program to find the roots of a quadratic equation.

(c) What are the three loop control statements available in C ? Write a comparison statement of the three.

(d) What is an array ? What are the different types of array ? Explain selection sorting algorithm to sort n numbers in ascending order.

(e) Explain with examples different types of functions.

4. What is the use of 'if-else' and 'nested if-else' statement ? Write down their formats. Write a C program to find biggest of three numbers using if-else and nested if-else statement. 2+2+3+3=10

Or

Write a C program to read the marks scored by a student in semester examination and print grade point along with the comment using the following : 10

(i) percentage > 90, "O", "OUTSTANDING"

(ii) percentage > = 75 and < = 90, "A" "VERY GOOD"

(iii) percentage > = 60 and < 75, "B", "GOOD"

(iv) percentage > = 50 and < 60, "C", "FAIR"

(v) percentage > = 40 and < 50, "D", "PASS"

(vi) percentage < 40, "F", "FAIL"

5. Write a C program to solve the series

$$s = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} \ldots\ldots,$$

which is the expansion of sine series with x in radians. 10

Or

Write a C program to multiply two matrices.

6. Write a C program to sort n numbers using bubble sort. 10

Or

What are the uses of recursine function ? Write a C program using recursine function for factorial of a number to find ${}^nC_r$.

2+8=10