Total number of printed pages–20

3 (Sem–3/CBCS) MAT HG 1/2/RC

2023

MATHEMATICS

(Honours Generic/Regular)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HG–3016 /MAT–RC–3016

(Differential Equation)

OPTION–B

Paper : MAT–HG–3026

(Linear Programming)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION–A

Paper : MAT–HG–3016 / MAT–RC–3016

(Differential Equation)

Answer **either** *in English* **or** *in Assamese.*

1. Answer the following questions : 1×10=10

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ

(a) Define order and degree of an ordinary differential equation.

সাধাৰণ অৱকল সমীকৰণৰ ক্ৰম আৰু ঘাতৰ সংজ্ঞা লিখা।

(b) What do you mean by an ordinary differential equation ? Give *one* example.

সাধাৰণ অৱকল সমীকৰণ বুলিলে কি বুজা ? এটা উদাহৰণ দিয়া।

(c) Define exact differential equation.

যথাৰ্থ অৱকল সমীকৰণৰ সংজ্ঞা লিখা।

(d) Obtain the differential equation of family of parabolas given by $y^2 = 4ax$.

$y^2 = 4ax$ অধিবৃত্তৰ পৰিয়ালটোৰ অৱকল সমীকৰণটো গঠন কৰা।

(e) Write the condition of exactness of an ordinary differential equation.

এটা সাধাৰণ অৱকল সমীকৰণৰ যথাৰ্থতাৰ চৰ্ত লিখা।

(f) Find the integrating factor of

$$\frac{dy}{dx} + \frac{y}{x} = \cos x .$$

$\frac{dy}{dx} + \frac{y}{x} = \cos x$, ৰ অনুকলন গুণক নিৰ্ণয় কৰা।

(g) Define orthogonal trajectory of a family of curve.

এটা বক্ৰ পৰিয়ালৰ লাম্বিক প্ৰক্ষেপপথৰ সংজ্ঞা লিখা।

(h) Write the complementary function of $\left(D^2 + 4\right)y = x^2$.

$\left(D^2 + 4\right)y = x^2$ অৱকল সমীকৰণটোৰ পৰিপূৰক ফলনটো লিখা।

(i) Write the general form of a linear differential equation of n^{th} order.

এটা n মাত্ৰাৰ ৰৈখিক অৱকল সমীকৰণৰ সাধাৰণ ৰূপটো লিখা।

(j) If $y_1 = sin2x$ and $y_2 = cos2x$, then find the Wronskian of $y_1(x)$ and $y_2(x)$.

যদি $y_1 = sin2x$ আৰু $y_2 = cos2x$, তেন্তে $y_1(x)$ আৰু $y_2(x)$ ৰ Wronskian নিৰ্ণয় কৰা।

2. Answer the following questions : 2×5=10

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ

(a) Determine the particular integral of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = sin2x.$$

$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = sin2x$ অৱকল সমীকৰণটোৰ বিশেষ অনুকলন নিৰ্ণয় কৰা।

(b) Derive the orthogonal trajectory of $xy = a^2$.

$xy = a^2$, ৰ লাম্বিক প্ৰক্ষেপপথ নিৰ্ণয় কৰা।

(c) Find the integrating factor of the differential equation

$$\left(x^2y - 2xy^2\right)dx - \left(x^3 - 3x^2y\right)dy = 0$$

$\left(x^2y - 2xy^2\right)dx - \left(x^3 - 3x^2y\right)dy = 0$ অৱকল সমীকৰণটোৰ অনুকলন গুণক নিৰ্ণয় কৰা।

(d) Solve : $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

সমাধান কৰা ঃ $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

(e) Solve : $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

সমাধান কৰা ঃ $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

3. Answer the following : ***(any four)*** 5×4=20

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ ***(যিকোনো চাৰিটা)***

(a) Solve : $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2$

সমাধান কৰা ঃ $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2$

(b) Find the orthogonal trajectories of the series of hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$.

$x^{2/3} + y^{2/3} = a^{2/3}$, পৰিয়ালটোৰ লাম্বিক প্ৰক্ষেপপথ নিৰ্ণয় কৰা।

(c) Solve the simultaneous linear differential equations $\frac{dx}{dt} = -py$ and $\frac{dy}{dt} = px$ and show that the point (x, y) lies on a circle.

$\frac{dx}{dt} = -py$ আৰু $\frac{dy}{dt} = px$; অৱকল সমীকৰণটো সমাধান কৰা আৰু দেখুওৱা যে (x, y) বিন্দুটো এটা বৃত্তত থাকিব।

(d) Solve by reducing to exact differential equation

$$xydx + (2x^2 + 3y^2 - 20)dy = 0$$

$xydx + (2x^2 + 3y^2 - 20)dy = 0$ সমীকৰণক যথাৰ্থ অৱকল সমীকৰণলৈ সমানীত কৰি সমাধান কৰা।

(e) Solve the Bernoulli's equation :

$$x\frac{dy}{dx} + y = y^2 \log x$$

বাৰ্নোলীৰ সমীকৰণটো সমাধান কৰা :

$$x\frac{dy}{dx} + y = y^2 \log x$$

(f) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$, given that $y = x^2$ is one of the solution.

$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ অৱকল সমীকৰণটো সমাধান কৰা, য'ত সমীকৰণটোৰ এটা সমাধান $y = x^2$.

4. Answer the following : ***(any four)*** 10×4=40

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া : ***(যিকোনো চাৰিটা)***

(a) Solve by the method of variation of parameter : $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

প্ৰাচল বিচৰণ পদ্ধতিৰে সমাধান কৰা :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

(b) Solve : $\dfrac{d^4y}{dx^4} - y = x\sin x$

সমাধান কৰা ঃ $\dfrac{d^4y}{dx^4} - y = x\sin x$

(c) Solve : $\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + y = 0$

$$\frac{dy}{dt} + 5x + 3y = 0$$

সমাধান কৰা ঃ $\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + y = 0$

$$\frac{dy}{dt} + 5x + 3y = 0$$

(d) Solve the exact differential equation :

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$

যথাৰ্থ অৱকল সমীকৰণটো সমাধান কৰা ঃ

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$

(e) Solve by reducing to normal form

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + \left(4x^2 - 1\right)y = -3e^{x^2}\sin 2x$$

নৰ্মাল ৰূপলৈ সমানীত কৰি সমাধান কৰা ঃ

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + \left(4x^2 - 1\right)y = -3e^{x^2}\sin 2x$$

(f) Show that the term $\dfrac{1}{x\left(x^2 - y^2\right)}$ is an integrating factor of the differential equation $\left(x^2 + y^2\right)dx - 2xy\,dy = 0$ and hence solve it.

দেখুওৱা যে $\left(x^2 + y^2\right)dx - 2xy\,dy = 0$ সমীকৰণৰ এটা অনুকলন গুণক $\dfrac{1}{x\left(x^2 - y^2\right)}$ আৰু সমাধান কৰা।

(g) Solve the equation, $4y = x^2 + p^2$, where

$$p \equiv \frac{dy}{dx}$$

সমাধান কৰা ঃ $4y = x^2 + p^2$, যত $p \equiv \frac{dy}{dx}$

(h) Discuss the method of solving a Bernoulli's equation of the form $\frac{dy}{dx} + Py = Qy^n$; where P and Q are constants as function of x.

এটা $\frac{dy}{dx} + Py = Qy^n$ ৰূপৰ বাৰ্নোলীৰ সমীকৰণ সমাধান কৰাৰ পদ্ধতি আলোচনা কৰা, য'ত P আৰু Q হৈছে ধ্ৰুৱক বা x ৰ ফলন।

OPTION–B

Paper : MAT–HG–3026

(Linear Programming)

1. Answer the following questions : *(Choose the correct answer)* 1×10=10

(a) A basic feasible solution whose variables are

(i) degenerate

(ii) non-degenerate

(iii) non-negative

(iv) None of the above

(b) The inequality constraints of an LPP can be converted into equation by introducing

(i) negative variables

(ii) non-degenerate B.F.

(iii) slack and surplus variables

(iv) None of the above

(c) A solution of an LPP, which optimize the objective function is called

(i) basic solution

(ii) basic feasible solution

(iii) optimal solution

(iv) None of the above

(d) Given a system of m simultaneous linear equations in n unknowns $(m<n)$ the number of basic variables will be

(i) m

(ii) n

(iii) $n-m$

(iv) $n+m$

(e) A simplex in n-dimension is a convex polyhedron having

(i) $n-1$ vertices

(ii) n vertices

(iii) $n+1$ vertices

(iv) None of the above

(f) At any iteration of the usual simplex method, if there is at least one basic variable in the basis at zero level and all $z_j - c_j \geq 0$ the current solution is

(i) infeasible

(ii) unbounded

(iii) non-degenerate

(iv) degenerate

(z_j, c_j having usual meaning)

(g) Let $X = \{x_1, x_2\} \subset \mathbb{R}^2$. Then the convex hull $C(X)$ of X is

(i) $\{\lambda x_1 + (1-\lambda) x_2 : \lambda \geq 1\}$

(ii) $\{\lambda x_1 + (1-\lambda) x_2 : \lambda \leq 0\}$

(iii) $\{\lambda x_1 + (1-\lambda) x_2 : 0 < \lambda < 1\}$

(iv) None of the above

(h) For given linear programming problem, if z is an objective function

(i) Max $z = -$ Min z

(ii) Max $z =$ Min $(-z)$

(iii) Max $(-z)$ = Max z

(iv) None of above

(i) The set $\{(x_1, x_2) : x^2_1 + x^2_2 \leq 1\}$ is a

(i) open set

(ii) closed set

(iii) neither open nor closed

(iv) open and closed both

(j) In linear programming problem

(i) objective function, constraints and variables are all linear

(ii) only objective function to be linear

(iii) only constraints are to be linear

(iv) only variables are to be linear

2. Answer the following : 2×5=10

(a) A hyperplane is given by the equation $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$, find in which half space do the point (–6, 1, 7, 2) lie.

(b) Prove that $x_1 = 2$, $x_2 = -1$ and $x_3 = 0$ is a solution but not a basic solution to the system of equations

$$3x_1 - 2x_2 + x_3 = 8$$

$$9x_1 - 6x_2 + 4x_3 = 24$$

(c) Write the dual of the following primal problem :

Minimize $Z = 3x_1 + 5x_2$

subject to $3x_1 + 5x_2 = 12$

$$4x_1 + 2x_2 = 10$$

with $x_1, x_2 \geq 0$

(d) In a two-person Zero-sum game, the pay-off matrix is given by

		B: I	II	III
A	I	6	8	6
	II	4	12	2

Find its saddle points.

(e) Show that the linear function

$Z = C\,X, X \in \mathbb{R}^n$, $C \in \mathbb{R}$ is a convex function.

3. Answer ***any four*** of the following : 5×4=20

(a) Solve graphically the following LPP :

Max. $Z = 5x_1 + 7x_2$

subject to $x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

$x_1, x_2 \ge 0$

(b) Find all basic feasible solutions of the system of equations

$x_1 + 2x_2 + 3x_3 + 4x_4 = 7$

$2x_1 + x_2 + x_3 + 2x_4 = 3$

(c) Prove that the set of all convex combinations of a finite number of points $x_1, x_2, x_3, \ldots\ldots, x_n$ is a convex set.

(d) Prove that the dual of a dual is a Primal problem itself.

(e) Solve the following transportation problem using North-West corner method whose cost matrix is given below :

Source	D_1	D_2	D_3	D_4	Supply
S_1	7	10	14	8	30
S_2	7	11	12	6	40
S_3	5	8	15	9	30
Demand	20	20	25	35	

(f) The pay-off matrix of a game is given below. Find the solution of the game to A and B.

		B				
		I	*II*	*III*	*IV*	*V*
A	*I*	−2	0	0	5	3
	II	3	2	1	2	2
	III	−4	−3	0	−2	6
	IV	5	3	−4	2	−6

4. Answer ***any four*** questions : 10×4=40

(a) Old hens can be bought for Rs. 2 each but young ones cost Rs. 5 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week, each being worth 30 paise. A hen costs Re. 1 per week to feed. If I have only Rs. 80 to spend for hens, how many of each kind shall I buy to give a profit of more than Rs. 6 per week, assuming that I can not house more than 20 hens? Formulate the LPP and solve by graphical method.

(b) Prove that if either the primal or the dual problem of an LPP has a finite optimal solution, then the other problem also has a finite optimal solution. Furthermore, the optimal values of the objective function in both the problems are the same, i.e.

$$\text{Max } Z_x = \text{Max } Z_w$$

(c) Solve the following assignment problem :

Projects

Engineer	A	B	C	D
I	12	10	10	8
II	14	Not suitable	15	11
III	6	10	16	4
IV	8	10	9	7

(d) Use simplex method to solve the LPP

Max $Z = 4x + 10y$

subject to the constraints

$$2x + y \le 50$$

$$2x + 5y \le 100$$

$$2x + 3y \le 90$$

$$x, y \ge 0$$

(e) Use the two-phase simplex method to solve Max $Z = 5x_1 - 4x_2 + 3x_3$

subject to the constraints

$$2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \le 76$$

$$8x_1 - 3x_2 + 6x_3 \le 50$$

$$x_1, x_2, x_3 \ge 0$$

(f) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

(g) If in an assignment problem, a constant is added or subtracted to every element of a row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimizes the total cost for the other matrix.

(h) (i) What is game theory? 2

(ii) Describe a two-person zero-sum game. Also mention *any two* basic assumptions in it. 4

(iii) Explain the following terms

Optimal strategy, Pay-off matrix. 2+2=4
