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1(Sem-1)MATH

2024

MATHEMATICS

Paper: MATM010104 / MATH010104

(CLASSICAL ALGEBRA)

Full Marks: 60 *Time: $2\frac{1}{2}$ hours*

The figures in the margin indicate full marks for the questions:

1. Answer the following questions: *1X7=7*

a) What is the sum of all the n^{th} roots of unity?

b) Mention the range of the exponential function of a complex number z.

c) If $a \neq 0$, then what is the sum of the product of the roots of the equation $ax^3 + bx^2 + cx + d = 0$ taken two roots at a time?

d) How many real roots are there in the equation

$$x^4 + 2x^2 + 3x - 1 = 0 ?$$

e) The rank of the matrix $\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}$ is

i. 0
ii. 1
iii. 2
iv. None of these

f) The diagonal elements of a skew-symmetric matrix are always ________ (Fill in the blank)

g) If a matrix A is reduced to an echelon form E by row operations, then rank of A is equal to the number of non-zero rows in E. (Write True or False)

2. Answer the following questions: *2X4=8*

a) If n is an integer, then show that $(1 + i)^n + (1 - i)^n = 2^{(\frac{n}{2}+1)} \cos\frac{n\pi}{4}$

OR

For $\in \mathbb{C}$, prove that $\cosh^2 z - \sinh^2 z = 0$

b) Show that the equation $x^6 - x + 6 = 0$ has no real root.

c) If α, β, γ be the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, then find the value of $\sum \alpha^2$

d) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$

OR

If A and B are symmetric matrices of same order, then prove that AB is symmetric if AB=BA.

3. Answer any three questions: *5X3=15*

a) Use De Moivre's theorem to prove that $\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

b) Prove that the sum of the 99th powers of the roots of $x^7 - 1 = 0$ is zero.

c) Apply Descarte's rule of signs to examine the nature of the roots of the equation

$$x^6 + x^4 + x^2 + x + 3 = 0$$

d) The roots of the equation $x^3 + ax^2 + bx + c = 0$ are α, β, γ. Find the equation whose roots are $\alpha\beta - \gamma^2, \beta\gamma - \alpha^2, \gamma\alpha - \beta^2$

e) Reduce the following matrix M to row echelon form and determine its rank.

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 8 \\ 2 & 6 & 0 \\ 1 & 2 & 5 \\ 3 & 8 & 6 \end{pmatrix}$$

4. Answer any three questions: *10X3=30*

a) (i) State and prove De-Moivre's theorem for rational indices.

1+4=5

(ii) If $n \in \mathbb{N}$, then show that $\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos(\frac{n\pi}{2} - n\theta) + i\sin(\frac{n\pi}{2} - n\theta)$

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b) Solve the following equations by using Cardon's method:

4+6=10

(i) $x^3 - 12x + 8 = 0$

(ii) $x^3 - 3x - 2\cos A = 0 \quad (-\pi < A \leq \pi)$

c) If the biquadratic equation $a_0x^4 + 4a_1x^3 + 6a_2x^2 + 4a_3x + a_4 = 0$ has two distinct pairs of equal roots, prove that two roots of the Euler's cubic are zero. Deduce that the equal roots are $\frac{-a_1 \pm \sqrt{3(a_1^2 - a_0a_2)}}{a_0}$ *10*

d) Determine the general solution for the following system of equations: *10*

$$2x + y + z = 0$$
$$4x + 2y + z = 0$$
$$6x + 3y + z = 0$$
$$8x + 4y + z = 0$$

e) Determine the reduced row echelon form of the following matrix, find its rank and then express each non-basic column in terms of the basic columns. *7+1+2=10*

$$\begin{pmatrix} 2 & -4 & -8 & 6 & 3 \\ 0 & 1 & 3 & 2 & 3 \\ 3 & -2 & 0 & 0 & 8 \end{pmatrix}$$